Vectors

*Mathematics*, 2016

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A vector in the Cartesian plane is an ordered pair (*a, b*) of real numbers. This is the mathematician's concise definition of a two-dimensional vector. Physicists and engineers like to develop this concept a bit more for the purpose of applying vectors in their disciplines, whereas in geometry, a vector simply describes a translation (a units left or right and b units up or down). Thus, they like to think of the mathematician's ordered pairs as representing displacements, velocities, accelerations, forces, and the like. Since such things have magnitude and direction, they like to imagine vectors as arrows in the plane whose magnitudes are their lengths and whose directions are the directions in which the arrows are pointing. The two small dark arrows shown in part (a) of the drawing below form the point of departure in the study of vectors in the plane.

These are called the unit basis vectors, or unit vectors. From them, all other vectors arise. To the mathematician, they are simply the ordered pairs (1, 0) and (0, 1). To the physicist, they really are the arrows extending from the origin to the points (1, 0) and (0, 1). The horizontal one is commonly named **i**, and the vertical one is commonly named **j**. To say that all other vectors in the plane arise from these two means that all vectors are linear combinations of **i** and **j**. For instance, the mathematician's vector (2, 3) is 2(1, 0) + 3(0, 1), while the physicist's arrow with horizontal displacement of 2 units and vertical displacement of 3 units is more compactly written as 2**i**+ 3**j**. It is represented in part (b) below.
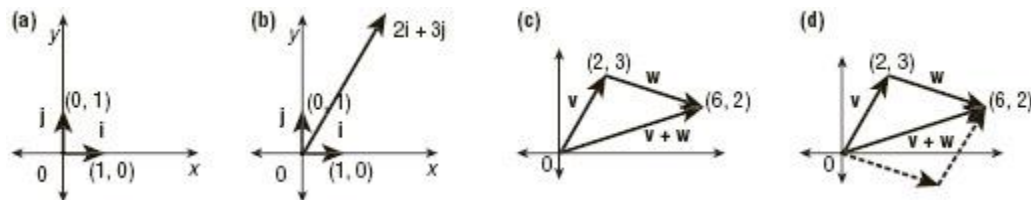
In the vocabulary of physics, the end of the arrow with the arrowhead is called the head of the vector and the end without the arrowhead is called the tail. The tail does not necessarily have to be at the origin as shown in the drawing. When it is, the vector is said to be in standard position, but any arrow with horizontal displacement 2 and vertical displacement 3 is regarded by physicists as 2**i** + 3**j**, as shown in part (b) of the drawing above. The 2 and 3 are called horizontal and vertical components, respectively, of the vector 2**i** + 3**j**.
By definition, if

v = (*a, b*) = *a***i** + *b***j**
and

w = (*c, d*) = *c***i** + *d***j**,
then

v+w = $(a + c, b + d)$ = $(a + c)\mathbf{i} + (b + d)\mathbf{j}$.

This definition of addition for vectors leads to a convenient geometrical interpretation. In part (c) of the drawing on the previous page,

v = 2**i** + 3**j** and

w = 4**i** − 1**j**.

Then **v** + **w** = 6**i** + 2**j**.

Physicists call their convention for adding geometric vectors head-totail addition. Notice in part (d) of the drawing on the previous page that v is in standard position and extends to the point (2, 3). If the tail of w is placed at the head of v, then the head of w ends up at (6, 2). Now if the arrow is drawn from the origin to (6, 2), the result is an appropriate representation of v + w in standard position. The vector v + w is usually called the resultant vector of this addition. Therefore, in general, the resultant of the addition of two vectors will be represented geometrically as an arrow extending from the tail of the first vector in the sum to the head of the second vector. This convention is often called the parallelogram law because the resultant vector always forms the diagonal of a parallelogram in which the two addend vectors lie along adjacent sides.

Applications of Vectors

As an example from physics, an object is being acted upon by two forces: a 30-pound (lb) force acting horizontally and a 40-lb force acting vertically. The physicist wants to know the magnitude and direction of the resultant force. The schematic diagram below represents this situation.
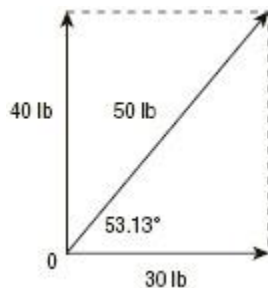

By the parallelogram law, the resultant of the two forces is the diagonal of the rectangle. The Pythagorean Theorem may be used to calculate that this diagonal has a length of 50, representing a 50-lb resultant force. The inverse cosine of 30/50 is 53.13°, giving the angle to the horizontal at which the resultant force acts on the object.

Consider another example in which vectors represent velocities. An airplane is attempting to fly due east at 600 mph (miles per hour), but a 50 mph wind is blowing from the northeast at a 45° angle to the intended due east flight path. If the pilot does not take corrective action, in what direction and with what velocity will the plane fly? The drawing on the next page is a vector representation of the situation.
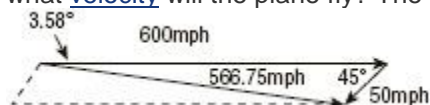
The vector representing the plane's intended velocity points due east and is labeled 600 mph, while the vector representing the wind velocity points from the northeast at a 45° angle to the line heading due east. These two vectors are positioned head to tail. Now the parallelogram law gives the actual velocity vector for the plane, the resultant vector, as the diagonal of the parallelogram with two sides formed by these two vectors. In this case, the Pythagorean Theorem may not be used, because the triangle formed by the three vectors is not a right triangle. Fortunately, some advanced trigonometry using the law of cosines * and law of sines * can be used to determine that the plane's actual velocity relative to the ground is 566.75 mph at an angle of 3.58° south of the line representing due east. In navigation, angles are typically measured clockwise from due north. Thus, a navigator might report that this plane was traveling 566.75 mph on a heading, or bearing, of 93.58°.

As another example of how mathematicians and physicists use vectors, a point is moving in the *xy*-coordinate plane so that it traces out some curve as the path of its motion. As the point moves along this curve, the *x*- and *y*-coordinates are changing as functions of time. In this case, $x = f(t)$ and $y = g(t)$. Now the mathematician will say that the position at any time *t* is $[f(), t\, g(t)]$ and that the position vector for the point is $R(t) = [f(t), g(t)] = f(t)\mathbf{i} + g(t)\mathbf{j}$. The physicist will say that the position vector R(*t*) is an arrow starting at the origin and ending with the head of the arrow at the point $[f(), t\, g(t)]$. (See the figure below.) It is now possible to define the velocity and acceleration vectors for this motion in terms of ideas from calculus, which are beyond the scope of this article.
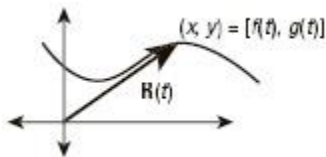
The mathematician's definition of a vector may be extended to three or more dimensions as needed for applications in higher-dimensional space. For example, in three dimensions, a vector is defined as an ordered triple of real numbers. Thus, the vector $\mathbf{R}(t) = [f(t), g(t), h(t)] = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ could be a position vector that traces out a curve in three-dimensional space.

*law of cosines for a triangle with angles A, B, C and sides a, b, c, $a2 = b2 + c2 - 2bc \cos A$
law of sines if a triangle has sides a, b, and c and opposite angles A, B, and C, then sin A/a = sin B/b = sin C/c
Stephen Robinson

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Bibliography

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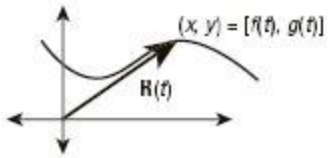
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R(t)
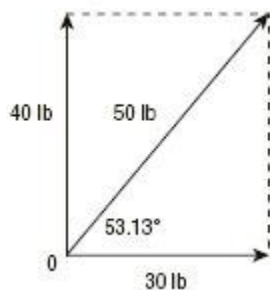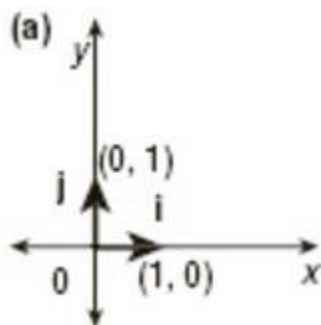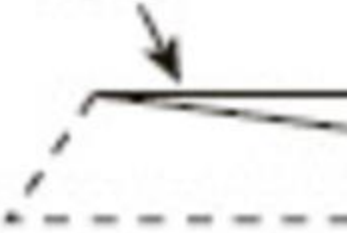
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